A Motorcycle or Bicycle as a Gyroscope (sort of)

by Dwight E. Neuenschwander

I was attending the national flat-track motorcycle races several years ago in Oklahoma City, watching the competitors run their qualifying laps, racing the clock individually to determine their positions at the start of the upcoming race. Coming out of the final turn and heading into the home stretch at 80 mph, the Triumph motorcycle ridden by great flat-tracker Don Castro suddenly went into a "speed wobble," which immediately grew in amplitude and threw Castro over the handlebars.

In flat-track racing the riders speed around a half-mile oval dirt track, hitting speeds of around 100 mph. The motorcycles are not the road-racing sports bikes that, on paved tracks, will round a corner at 150 mph, leaning at angles approaching 90 degrees; flat-trackers are stripped-down roaring heavy cruisers like Harley-Davidsons and Kawasaki Vulcans. To get through the turns at speed on dirt, the riders maintain a controlled power slide (Fig. 1).

Figure 1: Jared Mees and his Indian Scout FTR 750 motorcycle in a controlled power slide during a flat-track race. Photo courtesy of David Hoenig, Flat Trak Fotos.

In everyday riding the angle of the machine's lean from vertical and the angle turned by the front wheel are related, the front wheel turning in the same direction as the lean. In coming out of a normal turn those two angles approach zero together. Although oscillations can occur, if they do they are normally easily corrected by the rider. In a power slide the angles have opposite signs. To go into a dirt-track power slide for a left turn, upon entering the curve you throw your weight deliberately to the left while turning the handlebars hard to the right (see Fig. 1) and use the throttle to control the slide through the rear wheel's angular velocity. In coming out of the power slide to head down the straightaway, under some circumstances oscillations in the lean angle can amplify with astonishing rapidity into the dreaded "speed wobble." In Don Castro's case, he knew how to fall—wearing full leathers, gloves, boots and a sturdy helmet, he slid on his back, head first down the straightaway directly in front of the grandstands, with both hands on his helmet, seeming to enjoy his slide while his motorcycle tumbled after him. When they both stopped, Castro got up, dusted himself off, and pushed his bike back to the pits. Later that afternoon he competed in the race with the same bike and placed well. I don't know if Castro ever saw the equations that describe the steering and stability of a motorcycle, but he clearly knew how to skillfully *apply* the physics of riding!

The wheels of a motorcycle or bicycle ("bike" for either) make the machine a kind of gyroscope. A gyroscopic effect is apparent when a bicycle is pushed without a rider—it rolls in a straight line for a while, until it slows down and begins to lean over. Then the front wheel turns in the direction of the lean and the bike falls over. The same phenomenon appears in a rolling coin. Could a leaning moving bike be a precessing gyroscope, where a horizontal displacement of the center of mass (CM) creates a torque that turns the bike? When riding a motorcycle or bicycle I find that when I lean to the left or right while trying to keep the front wheel pointed straight ahead, the machine indeed moves in the direction of the lean, but ever so slowly—more of a drift instead of a deliberate turn (not good strategy for armadillo avoidance). The hypothesis that a leaning bike is a precessing gyroscope evidently forms a minor part of the story.*

Look carefully at the bike's front wheel and the fork connecting it to the handlebars. Visualize a line

Figure 2: Showing the castor distance δ on a 1994 Kawasaki Vulcan (top) and a 1962 J.C. Higgins Flightliner (bottom). Their front wheels have been deliberately set straight ahead. Photos by Dwight E. Neuenschwander.

extending through the fork's steering axis to the ground. The steering axis meets the ground *in front* of the tire's point of contact with the ground. The distance between the steering axis–ground intersection and the front tire's ground contact point is called the "castor" or "trail," here denoted δ (Fig. 2).

When the handlebars are turned, the tire contact point revolves around the steering axis because of the castor. You can feel this torque with an upright bike at rest: turn the handlebars back and forth and the bike frame behind the steering axis turns in the same direction through a smaller angle. The castor provides the lever arm that, with friction between the ground and tire, produces a torque that turns the bike. We will see that when the bike leans from the vertical (even when resting on the kickstand) the front wheel readily turns in the direction of the lean (see Fig. 3). For moving bikes, a relationship exists between the angle α through which the front wheel turns from the straight-ahead direction and the angle θ that the bike leans from the vertical. In our sign convention, both angles are positive for turns and lean to the rider's left.

Consider a bike moving upright and straight ahead on a horizontal road. Both wheels spin in the same vertical plane. Each wheel contributes a spin angular momentum to give a total angular momentum vector **L** that points horizontally to the rider's left. When I ride my bicycle or motorcycle, I imagine this angular momentum as an arrow sticking out of the machine's front axle. This vector grows longer when I speed up and shortens when I slow down.

To entertain the possibility of precession as a consequence of leaning, consider the torque due to the bike and rider's weight. Refer to the tire patch axis defined by the two points where the tires touch the road. When moving in a straight line without lean, the weight of the bike, the rider's CM, and the normal forces on the tires pass through the tire patch axis, producing zero torque about that axis. As a torqueless gyroscope the moving bike appears to be stable until the speed becomes too slow. But what happens when the bike leans?

Suppose the rider leans to the left, shifting the CM to the left of the tire patch axis. This induces a nonzero torque, $\mathbf{r} \times (m\boldsymbol{g})$ that points horizontally toward the rear of the bike (Fig. 3).

According to Newton's second law, in its rotational form a torque $\boldsymbol{\tau}$ produces a changing angular momentum given by the rate equation $\boldsymbol{\tau} = d\mathbf{L}/dt$. The angular momentum vector acquires a component $d\mathbf{L}$ that points toward the back of the bike—in the same direction as the torque. If this were the end of the story, as long as the bike leans the angular momentum vector **L** would rotate about a *vertical* axis—and the bike would turn—or rather slowly drift—to the left.

Trying to turn the bike by merely shifting your body weight off to the side produces a slow response, woefully inadequate for avoiding the car that suddenly pulls out in front of you.

Incidentally, we see why motorcyclists and bicyclists lean into a strong crosswind. If a strong wind comes from my left, the wind pressure exerted on me and the machine produces a clockwise torque (as viewed by an observer following me). To restore zero net torque, I must lean to the left.

A more effective way to turn left is to apply a small horizontal force *forward* on the left handlebar grip. The bike abruptly moves—counterintuitively—to the left. This is called *countersteering.* Motorcyclists and bicyclists do this even when they do not realize it. Try

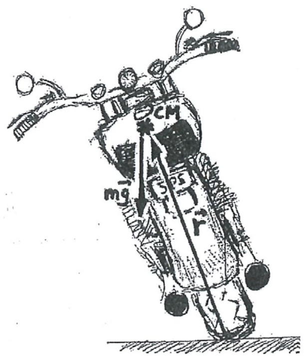

Figure 3: The torque produced by only the weight of the bike-rider center of mass (rider not shown) when leaning. The torque $\boldsymbol{r} \times (m\boldsymbol{g})$ points toward the back of the bike. All sketches by Dwight E. Neuenschwander.

it the next time you ride. When going straight, exert a gentle pressure forward on the left handlebar. The bike will respond quickly to the left. A preliminary way to think of it goes like this: when I gently apply a force forward on the left grip, I produce a torque about the steering axis that has a vertical downward component. To the horizontal angular momentum vector $\mathbf{L}$ is added an increment $d\mathbf{L}$ with a vertically downward component. Adding this $d\mathbf{L}$ to the original $\mathbf{L}$ tips the bike to the left, after which the center-of-mass offset contributes its torque.[1] Riding experience shows that with countersteering the bike turns deliberately and responsively, and the difference in response between countersteering and merely shifting one's weight arises from the interaction between the tires and the road. We must examine this in more detail. The physics of bike steering and stability is more complex to model than actually riding a bike—we learned to ride bicycles before we learned our multiplication tables!

The following analysis closely follows that of Lowell and McKell,[2] to which I hope to contribute a few value-added steps. Several other papers on this topic can also be recommended.[1,3-6] Figure 4 shows a schematic of the essential dimensions that concern us: a denotes the bike's wheelbase; Z is the point on the ground directly below the CM with the bike upright; b denotes the distance between Z and the rear tire's contact point with the ground; h is the height of the CM above the ground; and the castor distance δ is shown with an idealized vertical steering axis.

Consider the bike moving through a turn, its path the arc of a circle of radius R. From Fig. 5a, an overhead view of the bike, let α be the angle relative to

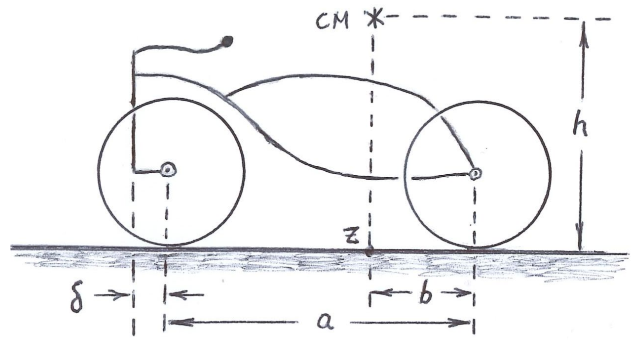

Figure 4: View of bike from the side, showing the distances a, b, h and δ.

the bike frame through which the front wheel is turned; let η be the angle relative to an arbitrary fixed direction through which the frame has turned as the bike moves along the arc; and let $\hat{\boldsymbol{n}}$ denote a unit vector normal to the bike's frame and pointing toward the center of curvature of the circular arc. From Fig. 5b, a view of the bike from behind it, let θ be the lean angle of the bike from the vertical. For dynamic variables let m denote the mass of the bike and rider and g the magnitude of the gravitational field.

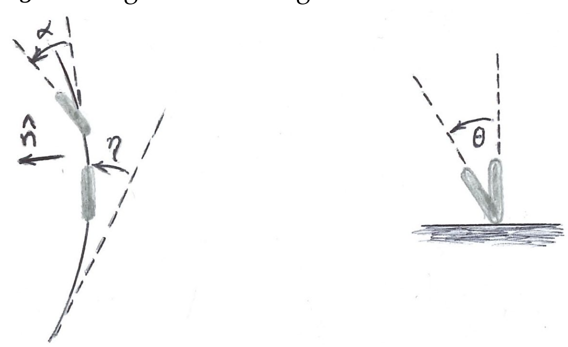

Figure 5. (a, left) View of bike from above, showing angles α and η and the unit vector $\hat{\boldsymbol{n}}$. (b, right) View of bike from behind it, showing the angle θ.

As the bike moves through the circular arc, its acceleration in the direction of $\hat{\boldsymbol{n}}$ comes from three displacements. If the bike were a point mass, it would undergo the centripetal acceleration

$$a_R = \frac{v^2}{R}, \tag{1}$$

where v denotes the bike's speed—which we assume to be constant throughout the turn. But as the bike leans the CM at height h gets displaced towards the arc's center of curvature, which contributes to the acceleration the amount

$$a_\theta = h\ddot{\theta}. \tag{2}$$

In addition, as the bike sweeps through the curve and its frame rotates through angle η, the CM acceleration also picks up the contribution

$$a_\eta = b\ddot{\eta} \tag{3}$$

in the same direction. Gathering all these contributions, the acceleration in the direction of $\hat{\boldsymbol{n}}$ is

$$a_n = \frac{v^2}{R} + h\ddot{\theta} + b\ddot{\eta}. \tag{4}$$

From the geometry of the bike and the definition of the radian, it follows that

$$v = R\dot{\eta} \tag{5}$$

and

$$\alpha = \frac{a}{R}. \tag{6}$$

Combining Eqs. (5) and (6) yields

$$v = \frac{a}{\alpha}\dot{\eta}. \tag{7}$$

From Eq. (7) we may write

$$\ddot{\eta} = \frac{v\dot{\alpha}}{a}, \tag{8}$$

and with this and Eq. (6), Eq. (4) becomes

$$a_n = \frac{v^2\alpha}{a} + h\ddot{\theta} + \frac{bv}{a}\dot{\alpha}. \tag{9}$$

The component of the gravitational force in the $\hat{\boldsymbol{n}}$ direction is $mg\sin\theta \approx mg\theta$, where we assume θ to be small. Neglecting friction (you *can* turn a bike on ice, but it's tricky—friction keeps the lean from going all the way to $\theta = \pi/2$ after the turn begins), the $\hat{\boldsymbol{n}}$ component of Newton's second law says

$$mg\theta = m\left(\frac{v^2}{a}\alpha + h\ddot{\theta} + \frac{bv}{a}\dot{\alpha}\right),$$

or, upon rearranging,

$$\ddot{\theta} + \frac{bv}{ha}\dot{\alpha} + \frac{v^2}{ha}\alpha - \frac{g}{h}\theta = 0. \tag{10}$$

Since leaning and turning the front wheel are related, if the angles are small, we might be justified in assuming a linear relation of the form

$$\alpha = k\theta, \tag{11}$$

where $k = const.$ If this is valid, then Eq. (10) becomes

$$\ddot{\theta} + \left(\frac{kbv}{ha}\right)\dot{\theta} + \frac{1}{h}\left(\frac{v^2k}{a} - g\right)\theta = 0. \tag{12}$$

If $k > {ga}/{v^2}$, this is the equation of a damped simple harmonic oscillator, raising the possibility of front-wheel oscillations. We consider these oscillations below.

With two spinning wheels giving a net angular momentum vector, one might think that gyroscopic effects would keep the bike stable, increasingly so with increasing speed. But in a series of experiments David Jones cleverly demonstrated that the gyroscopic effects are minor and not as essential to stability as one might assume.[6] Jones attached a third axle and wheel to the frame of a bicycle. The third wheel, parallel to the original two, did not touch the ground and could be made to spin in either sense. Its angular momentum added to that of the original wheels, changing the bike's gyroscope parameters to test the efficacy of gyroscopic action for stability. Jones found that the third wheel had negligible effects on stability. He then emphasized the importance of castor.

The effect of castor is easily demonstrated with a parked bike. With the kickstand down the bike leans over (most bikes to the left), and unless you deliberately set it otherwise the front wheel turns in the direction of the lean. When riding normally (no power slides), a nonzero lean angle θ turns the front wheel through a nonzero angle α with the same sign as θ. Conversely, turning the front wheel of a moving bike produces a lean, a point of physics exploited in countersteering: to move the machine to the left one pushes slightly *forward* on the left handlebar[7]—and to go the other way, a slight pull backwards on the left handlebar moves the machine to the right. Why? Pushing forward with a slight pressure on the *left* handlebar turns the front wheel to the *right* through a tiny angle $d\alpha < 0$, but because of the castor, the line of action of the force of friction acting sideways on the tire produces another torque, a restoring torque, that swings the front wheel to the left through a larger angle $\alpha > 0$ (Fig. 6b), and the bike leans into the direction of the turn—by Eq. (10), α and θ have the same sign after these angles stop changing. Restoring torques can produce oscillations, but consider what happens if the tire contact point sits behind the steering axis–ground intersection point (negative castor): then the torque is not a restoring torque, but a "repulsive" one (Fig. 6c).

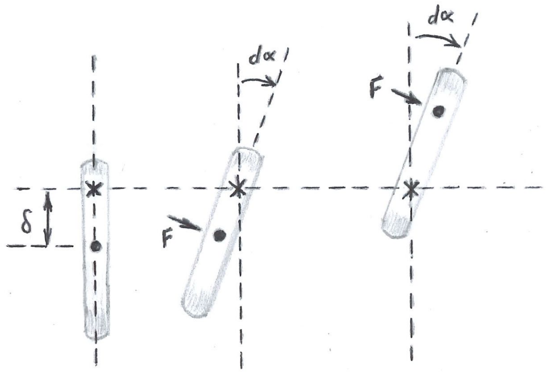

Figure 6: X denotes the point of intersection between the ground and the steering axis. The filled circle represents the tire contact point with the ground. δ is the castor. (a, left) $\alpha = 0$; (b, center) $d\alpha < 0$. F is the component of the frictional force perpendicular to the plane of the tire. $F\delta$ produces a restoring torque about X, which swings the tire to the left. (c, right) If X were behind the contact patch, then $F\delta$ would be a repulsive torque, not a restoring one. If δ were zero there would be very little lever arm for the tire's ground contact patch to turn the bike.

If I try to turn a moving bike left by cranking the handlebars toward the left initially, $d\alpha$ would change sign and the frictional torque in response would turn the bike to the right. If δ were zero, then friction would have very little lever arm to turn the bike at all, and if the steering axis intersects the ground behind the tire patch contact point, then a forward nudge on the left handlebar would cause a repulsive torque with positive feedback, steering the bike farther to the right (Jones reports such a bike to be the "nearest to being 'unrideable'"[6]).

The lean lowers the CM to produce a gravitational torque. That is crucial, so let's take a closer look. When the front wheel turns through the angle α, as seen in Fig. 7, the bike frame behind the steering axis turns through an angle φ,

$$\alpha\delta = a\varphi. \qquad (13)$$

This moves the CM in the direction of $\hat{\boldsymbol{n}}$ by the amount $b\varphi$ (Fig. 8a) so that by Eq. (13),

$$b\varphi = b\left(\frac{\alpha\delta}{a}\right). \qquad (14)$$

When the bike leans at angle θ, the CM drops the distance Δy (Fig. 8b), where

$$\Delta y = (b\varphi)\sin\theta$$
$$\approx \left(\frac{b\delta}{a}\right)\alpha\theta. \qquad (15)$$

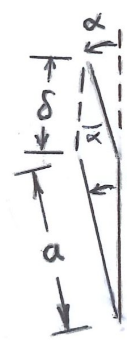

Figure 7: Turning the front wheel through angle α turns the bike frame behind the steering axis through the smaller angle φ in the same sense.

This drop corresponds to a decrease in the bike and rider's gravitational potential energy in the amount $mg\,\Delta y = (mgb\delta/a)\alpha\theta$. But a change in potential energy equals work, which in terms of torque τ about the steering axis is $\tau\alpha$ for the front wheel turning through angle α.Therefore $\tau\alpha = (mgb\delta/a)\alpha\theta$, and the torque associated with the lean of the bike is

$$\tau_{lean} = \left(\frac{mgb\delta}{a}\right)\theta. \qquad (16)$$

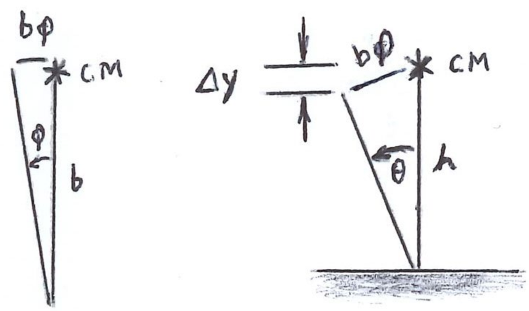

Figure 8: (a, left) Overhead view of the bike frame turning through angle φ. (b, right) View from behind the bike, showing the drop on the CM as the bike leans through angle θ.

Now let's add friction for a bike already in a turn, when θ and α are constants. With the front wheel turned from the straight-ahead direction, friction exerts a "sideways" force on the tires. Let F_f and F_b be the sideways component of the force of friction on the front and back tire, respectively, when the bike moves through a turn of radius R (Fig. 9). The $\hat{\boldsymbol{n}}$ direction component of $\mathbf{F} = m\mathbf{a}$ applied to the entire bike now gives

$$F_f + F_b = \frac{mv^2}{R}. \qquad (17)$$

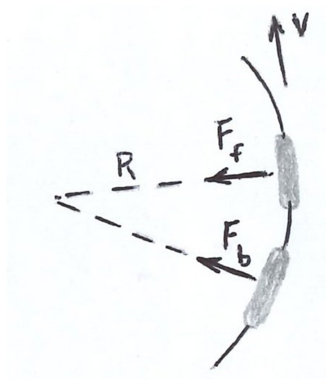

Figure 9: The sideways frictional forces acting on the bike tires as the bike moves through a turn of radius *R*.

In addition, the sum of torques about a vertical axis through the CM vanishes, so that

$$F_f\,(a-b) = F_b\, b. \qquad (18)$$

Solving Eqs. (17) and (18) for the frictional forces gives

$$F_b = \left(\frac{mv^2}{R}\right)\frac{a-b}{a} \qquad (19)$$

and

$$F_f = \left(\frac{mv^2}{R}\right)\frac{b}{a}. \qquad (20)$$

With the front wheel turned and the bike in the turn, the force F_f produces a frictional torque $\tau_{fric} = F_f\,\delta$ about the steering axis, in the opposite sense of τ_{lean}, so that

$$\tau_{fric} = -\left(\frac{mv^2}{R}\right)\frac{b\delta}{a},$$

which by Eq. (6), used to rewrite R in terms of the bike's wheelbase a, becomes

$$\tau_{fric} = -\frac{mv^2 b\alpha\delta}{a^2}. \qquad (21)$$

Newton's second law in rotational form says that net torque about the steering axis on the handlebars-fork-front wheel system—torques due to lean and friction—produces a change in the vertical component of angular momentum according to

$$\frac{mb\delta}{a}\left(g\theta - \frac{v^2\alpha}{a}\right) = \frac{d}{dt}\left(I_f\dot{\alpha} - I_o\omega\sin\theta\right) \quad (22)$$

where I_f denotes the moment of inertia of the handlebar-fork-front wheel assembly and I_o denotes the front wheel's moment of inertia about its axle with ω the wheel's angular velocity. Since Jones's experiments suggest that a gyroscopic effect does not dominate, if we ignore it and set $I_o = 0$, then Eq. (22) reduces to

$$\frac{mb\delta}{a}\left(g\theta - \frac{v^2\alpha}{a}\right) = I_f\ddot{\alpha}. \qquad (23)$$

Notice that $\ddot{\alpha} = 0$ if

$$\theta = \left(\frac{v^2}{ag}\right)\alpha, \qquad (24)$$

in which case the bike moves uniformly—in a straight line and upright if $\alpha = 0$, or in the arc of a circular path while leaning if $\alpha \neq 0$.

In the next installment we will consider bike stability and the effects of gyroscopic action.

Acknowledgment

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* This article offers a drastic revision of the Summer 2003 article, which was ridiculously over-simplified.

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7. In countersteering for a left turn, one can push the left handlebar forward, or pull the right handlebar backward, or both. Either way produces a clockwise initial torque, which is overcome by the counterclockwise torque due to sideways friction on the tire. For a right turn, push the right handlebar forward and/or pull the left handlebar backward. These are very subtle gentle pushes, not yanks, but the response is immediate.
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